Def: Let V be a V.S. over the same field F and non-empty subset S of V. If ~~W~~ is a V.S. over the same field F with the same op. on V

Then, ~~W~~ is a subsp of V.

$W(F) < V(F)$

Eg 1: $\mathbb{C}(\mathbb{R})$ is a V.S i.e. $(\mathbb{C}, +, \cdot)$

now $\mathbb{R} \subseteq \mathbb{C}$

$(\mathbb{R}, +, \cdot)$ is a V.S. itself.

$\therefore$ $\mathbb{R}(\mathbb{R})$ is a subsp. of $\mathbb{C}(\mathbb{R})$

Eg 2: $\mathbb{R}(\mathbb{Q})$ is a V.S.

$\mathbb{Q} \subseteq \mathbb{R}$, $\mathbb{Q}^c \subseteq \mathbb{R}$.

Which of the above is a V. Subspace of $\mathbb{R}(\mathbb{Q})$

✓(A) $\mathbb{Q}(\mathbb{Q})$ (C) Both

(B) $\mathbb{Q}^c(\mathbb{Q})$ (d) None

$\mathbb{Q}^c(\mathbb{Q})$ is not a V.S.

$\alpha = \sqrt{2}$, $\beta = -\sqrt{2}$ Then $\sqrt{2} + (-\sqrt{2}) = 0 \notin \mathbb{Q}^c$

$(\mathbb{Q}^c, +)$ is not a gp.

$\therefore$ $\mathbb{Q}^c(\mathbb{Q}) \not< \mathbb{R}(\mathbb{Q})$

NOTE 1: ① There are always two trivial subsp. of V(F) namely $\{0\}$ & V(F)

② V(F) is a improper subsp.